Math 45 SSM 2/e 7.2 Multiplying + Dividing Rational Expressions

(Day 1)

Objectives: 1) Multiply } Day 1
2) Divide } Day 2

To multiply rational expressions

step 1: Factor every numerator and every denominator completely.

step 2: Divide out common factors in numerators + denominators.

step 3: Leave final answer factored.

Multiply.

YES ① $\frac{5}{3} \cdot \frac{9}{10}$

$= \frac{5}{3} \cdot \frac{3 \cdot 3}{2 \cdot 5}$

"prime factors" for numbers are comparable to "factor completely" for polynomials.

$= \frac{\cancel{5}}{\cancel{3}} \cdot \frac{\cancel{3} \cdot 3}{2 \cdot \cancel{5}}$

$= \boxed{\frac{3}{2}}$

YES ② $\frac{x-3}{5} \cdot \frac{5x+35}{x^2-9}$

To Multiply Rational Expressions

step 1: factor completely

$= \frac{\cancel{(x-3)}}{\cancel{5}} \cdot \frac{\cancel{5}(x+7)}{\cancel{(x-3)}(x+3)}$

step 2: divide out or "cancel" factors that appear in both numerator and denominator

$= \boxed{\frac{(x+7)}{(x+3)}}$

step 3: Leave factored.

or $\boxed{\frac{x+7}{x+3}}$

If there's only one factor left in numerator or denominator, parentheses are optional.

YES ③ $\dfrac{p^2-9}{p^2+5p+6} \cdot \dfrac{p+2}{6-2p}$

Factor completely

1) $p^2-9 = (p-3)(p+3)$

2) p^2+5p+6 $\quad \begin{smallmatrix} & 6 & \\ 2 & \times & 3 \\ & 5 & \end{smallmatrix} \quad = (p+2)(p+3)$

3) $p+2$ already factored $(p+2)$

4) $6-2p = -2p+6$ standard form

$= -2(p-3)$ GCF w/ neg.

$$= \frac{\cancel{(p+3)}\cancel{(p-3)}}{\cancel{(p+2)}\cancel{(p+3)}} \cdot \frac{\cancel{(p+2)}}{(-2)\cancel{(p-3)}}$$

Cancel common factors

$$\frac{p+3}{p+3} \cdot \frac{p-3}{p-3} \cdot \frac{p+2}{p+2}$$

$$= \frac{1}{-2}$$

$$= \boxed{\frac{-1}{2}}$$

YES ④ $\dfrac{m^2-n^2}{10m^2-10mn} \cdot \dfrac{10m+5n}{2m^2+3mn+n^2}$

Factor completely.

1) $m^2-n^2 = (m-n)(m+n)$

difference of two squares

2) $10m^2-10mn = 10m(m-n)$

GCF $10m$

3) $10m+5n = 5(2m+n)$

GCF$=5$

4) $2m^2+3mn+n^2$ $\quad \begin{smallmatrix} & 2m^2n^2 & \\ 1mn & \times & 2mn \\ & 3mn & \end{smallmatrix}$

$= 2m^2 + 2mn + 1mn + n^2$

$= 2m(m+n) + n(m+n)$

$= (m+n)(2m+n)$

$$= \frac{\cancel{(m-n)}\cancel{(m+n)}}{10m\cancel{(m-n)}} \cdot \frac{5\cancel{(2m+n)}}{\cancel{(m+n)}\cancel{(2m+n)}}$$

$$= \frac{5}{10m}$$

$$= \boxed{\frac{1}{2m}}$$

YES (5) $\dfrac{4-z^2}{3z-2} \cdot \dfrac{3z^2+7z-6}{z^2+z-6}$

$= \dfrac{-\cancel{(z-2)}(z+2)}{\cancel{(3z-2)}} \cdot \dfrac{\cancel{(z+3)}\cancel{(3z-2)}}{\cancel{(z+3)}\cancel{(z-2)}}$

$= -(z+2)$

$= \boxed{-z-2}$

Final answer is not a fraction. Simplify by distribute.

If final answer is a fraction, leave factored

Factor completely.

1) $4-z^2$

$= -z^2+4$ standard form

$= -(z^2-4)$ GCF = -1

$= -(z-2)(z+2)$ diff of two squares

⇑ keep the (-1) in the answer.

2) $3z-2$ already factored

$= (3z-2)$

3) $3z^2+7z-6$ -18 9 -2 7

$= 3z^2+9z-2z-6$

$= 3z(z+3)-2(z+3)$

$= (z+3)(3z-2)$

4) z^2+z-6 -6 3 -2 1

$(z+3)(z-2)$

YES (6) $\dfrac{n^2-n-2}{4n^2-9} \cdot \dfrac{2n^2-n-6}{2n^2-5n+3}$

$= \dfrac{(n-2)(n+1)}{(2n-3)\cancel{(2n+3)}} \cdot \dfrac{(n-2)\cancel{(2n+3)}}{(2n-3)(n-1)}$

$= \dfrac{(n-2)(n+1)(n-2)}{(2n-3)(2n-3)(n-1)}$

$= \boxed{\dfrac{(n+1)(n-2)^2}{(n-1)(2n-3)^2}}$

Note: If the similar factors are both in the numerator (or both in the denominator) they cannot be canceled.

1) n^2-n-2 -2 -2 $+1$ -1

$(n-2)(n+1)$

2) $4n^2-9$ diff of sq.

$= (2n-3)(2n+3)$

3) $2n^2-n-6$ -12 -4 $+3$ -1

$= 2n^2-4n+3n-6$

$= 2n(n-2)+3(n-2)$

$= (n-2)(2n+3)$

4) $2n^2-5n+3$ 6 -3 -2 -5

$= 2n^2-3n-2n+3$

$= n(2n-3)-1(2n-3)$

$= (2n-3)(n-1)$

YES 7) $\dfrac{8n-8}{n^2-3n+2} \cdot \dfrac{2n^2+9n+10}{12}$

$= \dfrac{8\cancel{(n-1)}}{(n-2)\cancel{(n-1)}} \cdot \dfrac{(n+2)(2n+5)}{12}$

$= \dfrac{8}{12} \, \dfrac{(n+2)(2n+5)}{(n-2)}$

$= \boxed{\dfrac{2(n+2)(2n+5)}{3(n-2)}}$

Note: $(n+2)$ and $(n-2)$ are two different numbers.
For example: If $n=3$
$n+2 \Rightarrow 3+2=5$
$n-2 \Rightarrow 3-2=1$

1) $8n-8 = 8(n-1)$ GCF $=8$

2) n^2-3n+2 2, -2, -1, -3
$(n-2)(n-1)$

3) $2n^2+9n+10$ 20, 4, 5, 9

$= 2n^2+4n+5n+10$
$= 2n(n+2)+5(n+2)$
$= (n+2)(2n+5)$

4) 12.

NO like #3 8) $\dfrac{a^2-a-12}{a^2+2a-3} \cdot \dfrac{a-1}{12-3a}$

$= \dfrac{\cancel{(a-4)}\cancel{(a+3)}}{\cancel{(a+3)}\cancel{(a-1)}} \cdot \dfrac{\cancel{(a-1)}}{(-3)\cancel{(a-4)}}$

$= \dfrac{1}{-3}$

$= \boxed{\dfrac{-1}{3}}$

1) a^2-a-12 -12, -4, $+3$, -1
$= (a-4)(a+3)$

2) a^2+2a-3 -3, 3, -1, 2
$= (a+3)(a-1)$

3) $(a-1)$

4) $12-3a$
$= -3a+12$
$= -3(a-4)$ GCF $= -3$

No like #4

(9) $\dfrac{a^2+2ab+b^2}{6a^2+6ab} \cdot \dfrac{9a+3b}{3a^2+4ab+b^2}$

$= \dfrac{\cancel{(a+b)}\cancel{(a+b)}}{6a\,\cancel{(a+b)}} \cdot \dfrac{3\cancel{(3a+b)}}{\cancel{(a+b)}\cancel{(3a+b)}}$

$= \dfrac{3}{6a}$

$= \boxed{\dfrac{1}{2a}}$

1) $a^2+2ab+b^2$
$(a+b)(a+b)$
perfect square trinomial

2) $6a^2+6ab$
$= 6a(a+b)$
GCF $6a$

3) $9a+3b$
$= 3(3a+b)$

4) $3a^2+4ab+b^2$ 3 3 1 4
$= 3a^2+3ab+ab+b^2$
$= 3a(a+b)+b(a+b)$
$= (a+b)(3a+b)$

No like #7

(10) $\dfrac{5z+15}{2z} \cdot \dfrac{6}{z^2-2z-15}$

$= \dfrac{5\cancel{(z+3)}}{2z} \cdot \dfrac{6}{(z-5)\cancel{(z+3)}}$

$= \dfrac{5 \cdot 6}{2z(z-5)}$

$= \boxed{\dfrac{15}{z(z-5)}}$

reduce $\dfrac{30}{2} = 15$

1) $5z+15 = 5(z+3)$
GCF 5

2) $2 \cdot z$

3) 6.

4) $z^2-2z-15$ −15 −5 +3 −2
$(z-5)(z+3)$

No like #2

(11) $\dfrac{x+2}{7} \cdot \dfrac{7x+21}{x^2-4}$

$= \dfrac{\cancel{(x+2)}}{\cancel{7}} \cdot \dfrac{\cancel{7}(x+3)}{(x-2)\cancel{(x+2)}}$

$= \boxed{\dfrac{x+3}{x-2}}$

1) $(x+2)$

2) 7

3) $7x+21 = 7(x+3)$
GCF = 7

4) $x^2-4 = (x-2)(x+4)$
diff of squares

No Like #5

(12) $\dfrac{p^2-1}{2p-3} \cdot \dfrac{2p^2+p-6}{p^2+3p+2}$

$= \dfrac{\cancel{(p+1)}(p-1)}{\cancel{(2p-3)}} \cdot \dfrac{\cancel{(p+2)}\cancel{(2p-3)}}{\cancel{(p+2)}\cancel{(p+1)}}$

$= \dfrac{p-1}{1}$

$= \boxed{p-1}$

1) $p^2-1 = (p-1)(p+1)$
difference of squares

2) $(2p-3)$

3) $2p^2+p-6$ -12; 4, -3; 1

$= 2p^2+4p-3p-6$

$= 2p(p+2)-3(p+2)$

$= (p+2)(2p-3)$

4) p^2+3p+2 2; 2, 1; 3

$=(p+2)(p+1)$

7.2-4 Perform the indicated operation and simplify.

$$\frac{3x}{6x+3} \cdot \frac{8x+4}{3}$$

A. $\frac{4}{3}$

B. $\frac{4x}{9}$

C. $\frac{x}{3}$

✔ D. $\frac{4x}{3}$

standard form ✓

factor completely:

$$\frac{3 \cdot x}{3(2x+1)} \cdot \frac{4(2x+1)}{3}$$

cancel common factors:

$$\frac{3}{3} = 1 \quad \frac{2x+1}{2x+1} = 1$$

$$= \frac{x}{1} \cdot \frac{4}{3}$$

multiply:

$$= \boxed{\frac{4x}{3}}$$

7.2-13 **Perform the indicated operation and simplify.**

$$\overset{A}{\frac{m^2-9}{m^2+3m-18}}_{B} \cdot \overset{C}{\frac{m-3}{18+3m-m^2}}_{D}$$

- A. $-\frac{m-3}{m^2}$
- B. $\frac{m-3}{m^2-36}$
- C. $\frac{m+3}{m^2-36}$
- ✔D. $-\frac{m-3}{m^2-36}$

Factor all parts completely

A m^2-9
$=(m-3)(m+3)$

C $(m-3)$
already factored

B $m^2+3m-18$
$=(m+6)(m-3)$

-18: 6, -3; 3

D $18+3m-m^2$

write in standard form:

$-m^2+3m+18$

factor out -1

$-1(m^2-3m-18)$

$-(m-6)(m+3)$

-18: -6, $+3$; -3

This is where the negative comes from.

If you don't write in standard form, you get $(3+m)(6-m)$.

$6-m$ is <u>NOT</u> equivalent to $m-6$.

Cancel common factors

$$\frac{(m-\cancel{3})(m+\cancel{3})}{(m+6)(m-\cancel{3})} \cdot \frac{(m-3)}{(-1)(m-6)(m+\cancel{3})}$$

$$= \boxed{-\frac{(m-3)}{(m+6)(m-6)}} \leftarrow \text{best answer}$$

equivalent to

$$= \boxed{-\frac{(m-3)}{(m^2-36)}}$$

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7.2.45 **Perform the multiplication.**

$$\frac{3xy-2x^2-y^2}{x+y}\cdot\frac{y^2-x^2}{y^2-2xy}$$

$$\frac{3xy-2x^2-y^2}{x+y}\cdot\frac{y^2-x^2}{y^2-2xy}=\square$$ (Type your answer in factored form.)

$3xy-2x^2-y^2$

$= -2x^2+3xy-y^2$ ← A
gives lead ≠ 1

OR $= -y^2+3xy-2x^2$ ← B
gives lead = 1.

Usually we have terms organized as

x^2 — xy — y^2

OR as

y^2 — xy — x^2

purely one variable — mixed both variables — purely the other variable

Either A or B will work, but I'd choose B for two reasons

1 – It will put y-variables first, as they appear in most places in the problem

2 – It gives lead = 1 after factoring out −1.

$-y^2+3xy-2x^2$

$= -(y^2-3xy+2x^2)$ 2, -2, -1, -3

$= -(y-2x)(y-x)$

Problem gives:

$$\frac{-\cancel{(y-2x)}(y-x)}{\cancel{(y+x)}}\cdot\frac{(y-x)\cancel{(y+x)}}{y\cancel{(y-2x)}}$$

$$=\boxed{\frac{-(y-x)^2}{y}}$$

7.2.49 Perform the multiplication.

$$\textcircled{1}\ \frac{9n^2-49}{6n+12} \cdot \frac{15n^2-60}{3n^2+17n-56}$$

$$\frac{9n^2-49}{6n+12} \cdot \frac{15n^2-60}{3n^2+17n-56} = \frac{5(3n+7)(n-2)}{2(n+8)}$$ (Type your answer in factored form.)

YOU ANSWERED $\frac{5(3n+7)(n+2)}{2(n+8)}$

standard form ✓

Factor completely:

$$= \frac{(3n-7)(3n+7)}{6(n+2)} \cdot \frac{15(n+2)(n-2)}{(3n-7)(n+8)}$$

Cancel common factors

$$\frac{(3n-7)}{(3n-7)}, \frac{n+2}{n+2}$$

Reduce $\frac{15}{6} = \frac{5}{2}$

$$= \boxed{\frac{5(3n+7)(n-2)}{2(n+8)}}$$

① $9n^2-49$

$= (3n-7)(3n+7)$

② $6n+12$

$= 6(n+2)$

③ $15n^2-60$

$= 15(n^2-4)$

$= 15(n+2)(n-2)$

④ $3n^2+17n-56$

$(3n-7)(n+8)$ ✗ or guess + check

check

$3n^2+24n-7n-56$

$3n^2+17n-56.$

7.2.65 Perform the multiplication.

$$\frac{xy-ay+xb-ab}{xy+ay-xb-ab}\cdot\frac{3xy-3ay-3xb+3ab}{15b+15y}$$

①, ②, ③, ④

$$\frac{xy-ay+xb-ab}{xy+ay-xb-ab}\cdot\frac{3xy-3ay-3xb+3ab}{15b+15y}=\frac{(x-a)^2}{5(x+a)}$$

(Type your answer in factored form. Use integers or fractions for any numbers in the expression.)

4 terms ⇒ use grouping

① $xy-ay+xb-ab$

$y(x-a)+b(x-a)$

$(x-a)(y+b)$

② $xy+ay-xb-ab$

$y(x+a)-b(x+a)$

$(x+a)(y-b)$

③ $3xy-3ay-3xb+3ab$

GCF throughout

$= 3[xy-ay-xb+ab]$

$= 3[\,y(x-a)-b(x-a)]$

$= 3(x-a)(y-b)$

④ GCF

$15(b+y)$

$$=\frac{(x-a)\cancel{(y+b)}}{(x+a)\cancel{(y-b)}}\cdot\frac{\cancel{3}(x-a)\cancel{(y-b)}}{\underset{5}{\cancel{15}}\cancel{(b+y)}}$$

$b+y=y+b.$

$$=\boxed{\frac{(x-a)^2}{5(x+a)}}$$